
Laws of motion

Formulas and definition

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The momentum of a body $\vec{P} = m \vec{v}$ is a vector quantity.

It gives more information than the velocity. Its unit is kgms^{-1} or Ns and dimensional formula $M^1 L^1 T^{-1}$

Newton's second law of motion :

The time-rate of change in momentum of a body is equal to the resultant external force applied on the body and is in the direction of the external force.

$$F = \frac{d\vec{P}}{dt}$$

If mass is constant then $F = \vec{a}$

If mass varies but velocity constant rocket engines

Or person throwing weight out of trolley then

$$F = \vec{v} \frac{dm}{dt}$$

Here v is velocity of gas or liquid etc. constant with respect to rocket

Final velocity of rocket is

$$V = 2.303 \times v \times \log (m_0/m)$$

m_0 , m is initial and final mass of rocket and fuel respectively

The impulse of force is the product of force and the time for which it acts. when a large force acts for a very small time, it is difficult to measure F and Δt but the change in momentum can be measured, which is equal to the

impulse of force ($F \Delta t$)

Newton's third law of motion: " To every action there is always an equal and

opposite reaction."

Forces always act in pairs, and $\vec{F}_{AB} = -\vec{F}_{BA}$

F_{AB} is force on object A due to object B

F_{BA} is force on object B due to object A

Note that forces involved should be of same nature

Friction

Friction is produced due to the contact force between the surfaces in contact.

Direction of friction is opposite to external force, and if external force is zero friction is opposite to direction of velocity

It opposes the impending or the real relative motion.

Frictional force depends on normal force and coefficient of friction

If N is normal force, note that Normal force is perpendicular to surface. Thus on inclined plane gravitational force must be resolved

Coefficient of friction is dimension less quantity, it quantifies the frictional forces present between the surfaces.

Two types of coefficient of friction

Coefficient of Static Friction (μ_s): The coefficient of static friction measures the maximum frictional force that can be exerted before the surfaces start moving relative to each other.

Coefficient of Kinetic (or Dynamic) Friction (μ_k): The coefficient of kinetic friction measures the frictional force acting on the surfaces while they are sliding or moving relative to each other. Static frictional force

Static frictional force $f_s = \mu_s N$

If external force is equal to static friction, resultant force is zero and object is not set in motion, if external force is more than static friction it will set in motion. The coefficient of kinetic friction (μ_k) comes into play, which is generally lower in magnitude than the coefficient of static friction (μ_s).

kinetic frictional force $f_k = \mu_k N$

If external force is equal to kinetic friction object will move with constant velocity which will depend on force applied to overcome static friction

Turning or sudden change in direction

It happens when a centripetal force act on the object, they may be friction, gravitation, tension or normal force.

Formula for centripetal acceleration is

$$a = \frac{v^2}{r}$$

Formula for centripetal force is

$$F = m \frac{v^2}{r}$$

It's important to note that the centripetal force is a net force, which means it represents the vector sum of all the forces acting on the object to keep it in circular motion.

The maximum safe speed on level curved road is

Centripetal force is provided by friction

$$v_{max} = \sqrt{\mu_s R g}$$

R is radius of curvature, g is gravitational acceleration

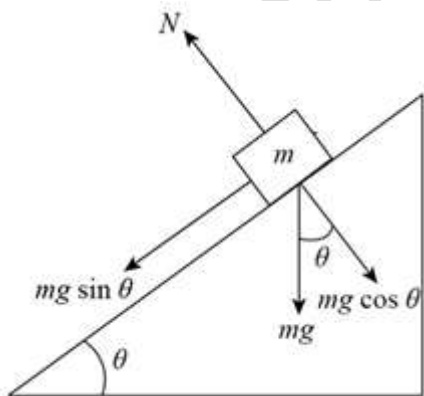
The maximum safe speed on a banked curved road is

Centripetal force is provided by friction and component of Normal force

$$v_{max} = \sqrt{rg \left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta} \right)}$$

Motion of object on inclined plane

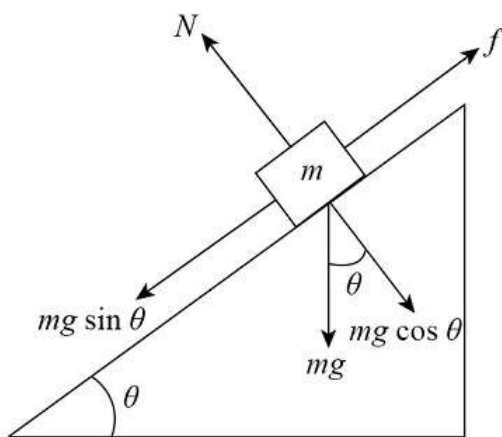
1) Frictionless surface : Component of g resolved



Note that acceleration down the inclined plane is $g \sin \theta$

Normal force is balanced by $mg \cos \theta$

2) Surface with friction



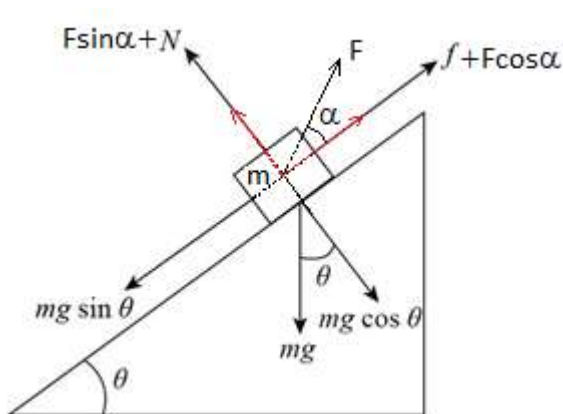
Down ward external force is $mg \sin \theta$ and frictional force is upward resultant force is

$$\text{Frictional force } f = \mu N = \mu mg \cos \theta$$

$$F = mg \sin \theta - f, \text{ or } F = mg \sin \theta - \mu mg \cos \theta$$

$$\text{Resultant acceleration } a = F/m = g \sin \theta - \mu g \cos \theta$$

3) Equilibrium of bodies on rough inclined plane External force F makes an angle of α with inclined plane



Equations at equilibrium

$$F \cos \alpha + f = mg \sin \theta \quad \text{here } f = \mu N$$

$$F \cos \alpha + \mu N = mg \sin \theta \quad \text{---(1)}$$

$$F \sin \alpha + N = mg \cos \theta \quad \text{---(2)}$$

$\alpha = 0^\circ$ if block is attached to rigid wall through string

If force along plane is not balanced object will move up or down resultant force

$$ma = F \cos \alpha + \mu N - mg \sin \theta$$

to solve further find value of N from (2)

If ma is positive object moving up, negative moving down

Pseudo Force

In non-inertial frame of reference due to acceleration one more additional force acting on a body in the opposite direction of acceleration of frame of reference is called pseudo force (F_p)

Example i) Inertia in a Bus or Train:

When a bus or train suddenly accelerates or decelerates, you might experience a backward or forward push respectively.

Elevator Acceleration: When an elevator starts moving up or down, you might feel heavier or lighter for a brief moment.

Formula

Climbing mountains or monkey climb on rope

(i) when a man of weight m climbs on the rope with acceleration a then tension in the rope is $T = m(g + a)$.

(ii) When man sliding down with acceleration a then tension in the rope is $T = m(g - a)$.

Lift/ elevator

Lift moving up with acceleration the pseudo force down ward

Apparent weight $M' = M(g+a)$

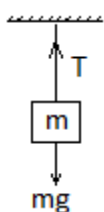
Lift moving down with acceleration the pseudo force upward Apparent weight $M' = M(g-a)$

Note if lift moves with constant velocity then $a = 0$

Direction of tension

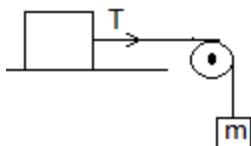
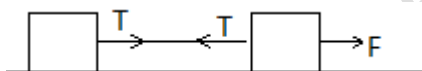
The tension in a rope or string is directed along the length of the rope, away from the object applying the force.

Vertical Tension:



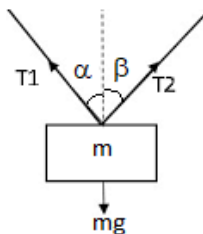
When an object is suspended by a rope or string, such as a hanging weight or a pendulum, the tension in the rope acts vertically upward. This tension counteracts the force of gravity pulling the object downward, maintaining equilibrium.

Horizontal Tension:



In situations where a rope is used to pull an object horizontally. The tension opposes any external forces applied to the object and causes it to accelerate or maintain motion.

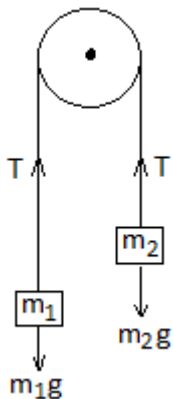
Angled Tension:



When a rope is at an angle with respect to the horizontal or vertical direction, the tension can be resolved into components perpendicular to the angle.

Pulley

For frictionless pulley



If $m_1 > m_2$, m_1 will move down and m_2 move up with acceleration 'a'

$$m_1 a = m_1 g - T$$

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) g$$

Tension

$$T = \frac{(2m_1 m_2)}{m_1 + m_2} g$$

In an idealized pulley system with a massless and frictionless string, the tension in the string remains constant throughout.